

Exam II , MTH 221, Fall 2015

Ayman Badawi

QUESTION 1. (i) Let $D = \{(a+b+2c,\ 3a-3b,\ a+2b+3c) \mid a,b,c \in R\}$. Then $dim(D) =$

a) 1 ~~b)~~ 2 c)3 d) None

(ii) Let A be a 2×2 matrix such that A is row-equivalent to $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$. Then the eigenvalues of A are :

a) $2, 4$ b)$\frac{1}{2}, \frac{1}{4}$ c) $\frac{1}{2}, 4$ ~~d)~~ None of the previous

(iii) Which of the following matrices with the given properties are (is) INVERTIBLE and diagnolizable:

~~a)~~A is 3×3, $C_A(x) = (x-3)^2(x-4)$, $E_3 = span\{(2,0,2),(0,1,4)\}$, and $E_4 = span\{(0,0,9)\}$

b) A is 2×2, $C_A(x) = (x-4)^2$ and $E_4 = span\{(0,7)\}$

c) A is 2×2, $C_A(x) = x(x-2)$, $E_0 = span\{(4,1)\}$, and $E_2 = span\{(0,5)\}$

d) a and b (e) b and c (f) a and c

(iv) Let $A = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & -1 & -4 \\ 0 & 0 & -3 & -12 \end{bmatrix}$. Then $N(A) =$

a) $span\{(0,0,1,4),(1,0,0,0)\}$ b) $span\{(0,2,0,0)\}$ ~~c)~~ $span\{(0,1,0,0),(0,0,-4,1\}$ d) None of the previous

(v) Let $A = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & -1 & -4 \\ 0 & 0 & -3 & -12 \end{bmatrix}$. Then $col(A)$

~~a)~~ Span $\{(0,1,0),(4,-4,-12)\}$ b) Span $\{(0,1,0),(1,0,0)\}$ c) span$\{(1,-1,-2)\}$ d) None of the previous

(vi) Let $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -6 \\ 0 & 1 & -5 \end{bmatrix}$. Then the eigenvalues of A are :

a) 0, -5 ~~b)~~ 0, -2, -3 c) 0, -6, -5 d) 1, -5, -6 e)None of the previous

(vii) Let D be a subspace of $R^{2\times 2}$ such that $dim(D) = 2$. Then a possibility for D is

a) $D = \{\begin{bmatrix} a+2b & 2a+4b \\ 0 & 0 \end{bmatrix} \mid a,b \in R\}$ b) $D = \{\begin{bmatrix} a+3 & 4a \\ b & 6b \end{bmatrix} \mid a,b \in R\}$

~~c)~~ $D = \{\begin{bmatrix} a+2b+c & 3a+6b \\ c & 0 \end{bmatrix} \mid a,b,c \in R\}$ d) $D = \{\begin{bmatrix} a+2b & 2a+4b \\ c & a+b \end{bmatrix} \mid a,b,c \in R\}$

(viii) Let A be a 2×2 matrix such that $A\begin{bmatrix} 1 \\ 9 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 9 \end{bmatrix}$ and $det(A) = 15$. Then Trace(A) =

a) 6 ~~b)~~ 8 c) 30 d) 10 e) Need more information.

(ix) Given $D = \{(a,b,c) \in R^3 \mid a+b = 0$ and $a+c = 0$, where $a,b,c \in R\}$ is a subspace of R^3. Then $D =$

a) $span\{(1,0,-1),(1,-1,0)\}$ ~~b)~~ $span\{(-6,6,6)\}$ c) $span\{(0,1,-1),(1,-1,0)\}$ d) R^3 e)None of the previous

(x) One of the following is a basis for P_3

~~a)~~$\{1+x^2, -x-x^2, x^2\}$ b)$\{1+x+x^2, -1-x-2x^2, 1+x+5x^2\}$ c) $\{5, x-3x^2, 10+3x-9x^2\}$ d) $\{10, x+3, 16+2x\}$ e) None of the previous is correct

(xi) Given $F = \{f(x) \in P_4 \mid f'(2) = 0\}$ is a subspace of P_4. Then a basis for F is

a) $\{x-2, x^2-4, x^3-8\}$. ~~b)~~ $\{x^2-4x,\ x^3-12x\}$ c) $\{x^3+x^2-16x,\ x^3-3x^2,\ x^2-4x\}$ d) None of the previous

(xii) One of the following is true:

a) $\{A \in R^{2\times 2} \mid det(A^T) = 0\}$ is a subspace of $R^{2\times 2}$ b) $\{(a,b,c) \in R^3 \mid a,b,c \in R\ \ and\ \ a+b+c-1 = 0\}$ is a subspace of R^3

~~c)~~ $\{(a^3, b, a^3) \mid a, b \in R\}$ is a subspace of R^3 d) $\{(a, 3a+b, -b) \mid a, b \geq 0\}$ is a subspace of R^3.

(xiii) Given that $S = \{A \in R^{2\times 2} \mid Trace(A) = 0\}$ is a subspace of $R^{2\times 2}$. Then $dim(S) =$

a) 4 ~~b)~~ 3 c) 1 d) 2

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Exam II, MTH 221, Spring 2015

Ayman Badawi

QUESTION 1. (10 points) Let $A = \begin{bmatrix} 1 & b & 4 \\ a & 3 & 1 \\ 4 & c & 0 \end{bmatrix}$. Given A is row-equivalent to $\begin{bmatrix} 0 & 0 & 2 \\ 2 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

(i) Find the values of b, a, c. **Trivial/ Ideas discussed in class**

(ii) Find a basis for the column space of A. **Trivial**

QUESTION 2. (10 points) Let $S = \{(a + b + 2c, 3a + 6c, 2a - b + 4c) \mid b, c \in R\}$. Is S a subspace of R^3? explain. If yes, find dim(A), find basis for A, and write A as a span of a basis. **Trivial/ basic question**

QUESTION 3. (10 points)

(i) Find a basis for P_4 such that each element in the basis is of degree 3. Show the work. **some thinking is involved here, so we need 4 INDEPENDENT polynomials each is of degree 3. As you translate to points in R^4 (assume polynomials are written in descending order according to their degree), so we need to form a matrix 4×4 such that all entries in the first column are 1 (to ensure getting polynomials each of degree 3). Now you stare at the matrix and choose the other entries so that when you change it to semi-echelon all rows survive. For example Take the matrix $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$. Now by staring at A, $det(A) \neq 0$. So all rows are independent (or change it to semi-echelon/ all rows will survive). Now translate back to polynomials: so $x^3, x^3 + x^2, x^3 + x, x^3 + 1$ is the desired basis**.

(ii) Let f_1, f_2, f_3, f_4 be polynomial in P_4 such that each is of degree 2. Show that f_1, f_2, f_3, f_4 are dependent.

This is supposed to be a trivial one, note that $f_1, ..., f_4$ are elements of P_3 as well. Since $dim(P_3) = 3$, every 4 elements in P_3 are dependent

QUESTION 4. (12 points) TYPICAL BASIC QUESTION/ SEE CLASS NOTES

a) Let $A = \{F \in R^{3\times3} \mid Rank(F) \leq 2\}$. Then A is not a subspace of $R^{3\times3}$. Why?

b) Let $A = \{(a, b, c) \mid a + 2b + c = 4\}$. Then A is not a subspace of R^3. Why?

c) Let $A = \{f(x) \in P_4 \mid f(2) = 0 \text{ or } f(3) = 0\}$. Then A is not a subspace of P_4. Why?

d) Let $A = \{F \in R^{3\times3} \mid det(A) = 0\}$. Then A is not a subspace of $R^{3\times3}$. Why?

QUESTION 5. a) **(10 points)** Let $F = \{A \in R^{2\times2}$ such that $A \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}\}$. Show that F is a subspace of $R^{2\times2}$. Then find a basis for F. **Done in class/ see your notes**

b) **(6 points)** Given L = $\{A, B, C, D\}$ is a basis for $R^{2\times2}$, where $A = \begin{bmatrix} 2 & 4 \\ -2 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -4 \\ 2 & -5 \end{bmatrix}$. Find C and D. Note that C and D are not unique. Show the work

Here is the idea: We need to form a matrix F, 4×4, with 4 independent rows. Of course the first two rows are A and B. Now stare at F add two more rows so that when you change F to the semi-echelon all rows survive. After you do that, translate each row of the two rows you added to an 2×2 matrix.

QUESTION 6. (12 points) Given A is a 2×2 matrix such that $3, -3$ are eigenvalues of A, $E_3 = span\{(4,2)\}$, and $E_{-3} = span\{(-2,0)\}$.

(i) Find the trace of A^{-1}. **basic/ see class notes**

(ii) Show that A^2 is a diagonalizable, i.e., find an invertible matrix W and a diagonal matrix D such that $W^{-1}A^2W = D$. **basic**

(iii) Show that A^T is diagnolizable, i.e., find an invertible matrix W and a diagonal matrix D such that $W^{-1}A^TW = D$. **Since** $F^{-1}AF = D$**,** $F^TA^T(F^{-1})^T = D^T$**. Now here** $W = (F^{-1})^T$**.**

(iv) Find a nonzero 2×4 matrix D such that $AD = 3D$. **by matrix multiplication, let** d_1, d_2, d_3, d_4 **be the columns of** D**. Then** $Ad_1 = 3d_1$**,** $Ad_2 = 3d_2$**, ...,** $Ad_4 = 3d_4$**. By staring at the question and knowing 3 is an eigenvalue of** A**/ all these columns must come from** E_3**. Choose all column of** D **from** E_3 **and we are done.**

QUESTION 7. (18 points) Let A be a 3×3 matrix such that $C_A(x) = x(x-5)^2$. Given $Nul(A) = \{(x_1, 2x_1, 0) | x_1 \in R\}$ and $Nul(5I_3 - A) = \{(3x_3, 0, x_3) | x_3 \in R\}$,

(i) find $det(A)$.

(ii) Is A diagnolizable? if yes, find a diagonal matrix D and an invertible matrix W such that $W^{-1}AW = D$. If no, explain. **No. Since** $dim(E_5) = 1$ **but** 5 **has multiplicaty 2. Note** $Nul(A) = Nul(-A) = E_0$ **and** $Nul(5I_3 - A) = E_5$

(iii) Find Trace of A. **Adding eigenvalues with multiplicaty. So 5 + 5 + 0 = 10**

(iv) Find $det(A - 2I_3)$. **We know eigenvalues of** $A - 2I_3$ **are 0 - 2, 5 - 2, 5 - 2. Multiply them we get** -18

(v) Let $D = A + 11I_3$. Find a nonzero point in R^3, say $v = (a,b,c)$, such that $D \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 11a \\ 11b \\ 11c \end{bmatrix}$ **Since** 0 **is an egivalue of** A **choose a nonzero point** v **in Nul(A) =** E_0**. Since** $Av = 0v = 0$**, we have** $Dv = (A + 11I_3)v = Av + 11v = 0 + 11v = 11v$**.**

(vi) Given $F = \begin{bmatrix} a & b & \pi \\ c & d & e \\ \sqrt{3} & 8 & f \end{bmatrix}$ such that $AF = 5F$. Find all entries of F. What is the rank of F. **The same idea as in V. So the column of** F **must come from** E_5**. So set the first column of** F**,** $(a, c, \sqrt{3}) = (3x_3, 0, x_3)$**. We get** $x_3 = \sqrt{3}$**, c = 0 and** $a = 3x_3 = 3\sqrt{3}$**, now do similar for column II and column III. Since all columns of** F **are coming from** E_5 **and** $dim(E_5) = 1$**, Rank(F) = 1**

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Exam II , MTH 221 , Spring 2012

Ayman Badawi

QUESTION 1. (19 questions, each = 2 points, Total of points = 38)

(i) let $A = \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix}$. The eigenvalues of A are

~~a)~~ -2, 1 b) 0, -1 c) 1, -1 d) None of the previous

(ii) Let A be a 2×2 matrix such that A is row-equivalent to $\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$. Then the eigenvalues of A :

a) Cannot be determined b) 2, 0 c) 0.5, 0 ~~d)~~ None of the previous

(iii) Let A as in the previous question. Then $N(A)$ =

a) $\{(0,0)\}$ ~~b)~~ span$\{(0,1)\}$ c) span$\{(2,0)\}$ d) None of the previous

(iv) One of the following matrices with the given properties is diagnolizable:

a)A is 3×3, $C_A(\alpha) = (2-\alpha)^2(3-\alpha)$, and $E_2 = span\{(2,4)\}$, $E_3 = span\{(3,3)\}$

~~b)~~ A is 2×2, $C_A(\alpha) = -\alpha(5-\alpha)$, $E_0 = span\{(0,1)\}$, $E_5 = span\{(3,0)\}$

c) A is 2×2, $C_A(\alpha) = \alpha^2$, $E_0 = span\{(4,1)\}$

d) None of the previous

(v) Given A is 4×4, $C_A(\alpha) = (2-\alpha)^3(5-\alpha)$. Then $det(A^T)$ =

a) 10 ~~b)~~ 40 c) $\frac{1}{10}$ d) $\frac{1}{40}$

(vi) Let A as above such that $A \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$. Then one of the following is true:

a) $A \begin{bmatrix} 1 \\ 0 \\ 1.5 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$

b) $A^{-1} \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1.5 \end{bmatrix}$

~~c)~~ (a) and (b) correct

d) $A^{-1} \begin{bmatrix} 1 \\ 0 \\ 1.5 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$

(vii) Given A is 2×2, 0 and 1 are eigenvalues of A such that $E_0 = \{(-x_2, x_2) \mid x_2 \in R\}$ and $E_1 = \{(0, x_2) \mid x_2 \in R$. THEN (JUST WRITE DOWN THE ANSWER) $A =$

(viii) Given A is 3×3, $C_A(\alpha) = (3-\alpha)^2(a-\alpha)$. B is 3×3 and B is similar to A such that $det(B) = 54$. Then $a =$

a) 18 b) 1/18 ~~c)~~ 6 d) Cannot be determined

(ix) Let $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ -1 & -2 & 0 & 1 \\ 1 & 3 & 1 & 1 \end{bmatrix}$. Then $N(A) =$

a) $\{(-2x_3 - 2x_4, x_3 + x_4, x_3, x_4) \mid x_3, x_4 \in R\}$

~~b)~~ $\{(2x_3, -x_3, x_3, 0) \mid x_3 \in R\}$

c) $\{(2x_3 + 2x_4, -x_3 - x_4, x_3, x_4 \mid x_3, x_4 \in R\}$ d) None of the previous.

(x) Let A as above then $N(A)$ (as span)

a) Span $\{(-2, 1, 1, 0), (-2, 1, 0, 1)\}$ ~~b)~~ Span$\{(-4, 2, -2, 0)\}$ c) span$\{(2, 1, 1, 0)\}$

d) Span$\{(2, -1, 1, 0), (2, -1, 0, 1)\}$ e) None of the previous

(xi) Let A as in Question ix. Column Space of A =

a) Span$\{(1, 0, 0), (0, 0, 1), (-2, 0, 1)\}$ ~~b)~~ Span $\{(1, -1, 1), (2, -2, 3), (0, 1, 1)\}$ c)Span$\{(1, 0, 0), (0, 0, 1), (-2, 1, 1)\}$
d) None of the previous

(xii) Let A as in Question ix. Then For each $B \in R^3$

~~a)~~ The system of linear equations $AX = B^T$ has infinitely many solutions

b) The system of linear equations $AX = B^T$ has unique solution

c) The system of linear equations $AX = B^T$ may and may not be consistent (it all depends on the selected B)

d) I need more information, you keep asking nonsense questions.

(xiii) One of the following is a subspace of P_3

~~a)~~ $\{(2a_1 + a_0)x^2 + a_1x + a_0 \mid a_0, a_1 \in R\}$. b) $\{a_2x^2 + x + a_0 \mid a_0, a_2 \in R\}$ c) $\{x^2 + a_1x + a_0 \mid a_0, a_1 \in R\}$

(xiv) Let D be a subspace of $R_{2\times2}$ such that $dim(D) = 2$. Then a possibility for D is

~~a)~~ $D = \{\begin{bmatrix} a+b & a \\ 0 & b \end{bmatrix} \mid a, b \in R\}$ b) $D = \{\begin{bmatrix} a+1 & a \\ b & 0 \end{bmatrix} \mid a, b \in R\}$

c) $D = \{\begin{bmatrix} a+b+c & a \\ c & b \end{bmatrix} \mid a, b, c \in R\}$ d) $D = \{\begin{bmatrix} a+2b & 2a+4b \\ 0 & a+2b \end{bmatrix} \mid a, b \in R\}$

(xv) Let $D = \{(a + b + 2c, a - b, a + 2b + 3c) \mid a, b, c \in R\}$. Then $dim(D) =$

a) 1 b) 3 ~~c)~~2 d) None

(xvi) Let D as in QUESTION xv. Then a basis for D

a) $\{(4, 0, 6)\}$ b) $\{(1, 1, 1), (1, 0, -1)\}$ ~~c)~~ $\{(1, 1, 1), (1, -1, 2)\}$ d) $\{(1, 1, 1), (1, -1, 2), (2, 0, 3)\}$

(xvii) One of the following points is in D (D is as in Question xv)

a) $(7, 1, 11)$ b) (-2, 0, 3) ~~c)~~ (1, 5, -1) d) (0, -2, -3)

(xviii) One of the following is a basis for P_3

~~a)~~ $\{x^2, x^2 + x, x^2 + 1\}$ b) $\{x^2, x^2 + 2x + 1, 2x + 1\}$ c) $\{1, 1 + x + x^2, -1 + x + x^2\}$ d) None of the previous

(xix) Given A is a 3×3 matrix and $C_A(\alpha) = (1 - \alpha)^2(3 - \alpha)$. Given $E_1 = span\{(1, 1, 1), (1, -1, 0)\}$ and $E_3 = span\{(0, 0, -1)\}$. Let Q be a 3×3 invertible matrix such that $Q^{-1}AQ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Then $Q =$

~~a)~~ $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$. b) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$. c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$. d) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$.

Second Exam MTH 221 , Fall 2011

Ayman Badawi

QUESTION 1. (Circle the correct answer, each = 2.5, total 17.5)

(i) One of the following set is equal to R^2:

a) $Span\{(2,1),(6,3)\}$ b) $\{(3,0),(2,4)\}$ ~~c)~~ $\{(a+2b,-4b) \mid a,b \in R\}$ d) $Span\{(4,6)\}$

(ii) One of the following is a subspace of R^3:

a) $\{(a,2a,b^2) \mid a,b \in R\}$. b)$\{(a+b,0,2+b) \mid a,b \in R\}$ c) $\{(3,0,a+b) \mid a,b \in R\}$ ~~d)~~ $\{(a,3b-a,b) \mid a,b \in R\}$

(iii) Let A be a particular matrix 3×4 such that $N(A) = \{(a_3, a_3+a_4, a_3, a_4) \mid a_3, a_4 \in R\}$. Then one of the following statement is true:

a) If the system of linear equations $AX = \begin{bmatrix} 2 \\ 3 \\ 4.2 \end{bmatrix}$ has a solution, then the solution is unique. ~~b)~~ There must be a point $B = (b_1, b_2, b_3) \in R^3$ such that the system $AX = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ has no solutions. c) The system $AX = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ has infinitely many solutions. d) None of the previous statements is correct.

(iv) Let A, $N(A)$ as in the previous question. Let B be a matrix 4×3 such that $Rank(B) = 2$ and $AB = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

Then one of the following is a possibility for B:

~~a)~~ $\begin{bmatrix} 1 & 3 & 0 \\ 1 & 3 & 2 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. b) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ -1 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ d) It is possible that B be as in (a) and as in (b) and as in (c). e) There is no way that we can determine a possibility for B since A is not completely determined.

~~(v) Let A, $N(A)$ as in (iii). Given that $A\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$. Then one of the following must be true:~~

~~a) $A\begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ b) $A\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ c) $A\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ d) The solution to $AX = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ is unique and hence $x_1 = 0, x_2 = 1, x_3 = 1, x_4 = 1$ is the only solution to the system $AX = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$~~

(vi) Let $F = Span\{(a+c) + (a+c)x + (a+b+2c)x^2 \mid a,b,c \in R\}$ be a subspace of P_3. Then $dim(F) =$

~~a)~~ 2 b) 3 c)1 d) Cannot be determined

(vii) Let $F = \{(a+c, a+c, a+b+2c) \mid a,b,c \in R\}$. Then $F =$

~~span~~$\{(1,1,1),(0,0,6)\}$ b) $Span\{(2,2,2),(-1,-1,-1)\}$ c) Span $\{(1,1,2)\}$ d) R^3

QUESTION 2. (Circle the correct answer, each = 2.5, total = 22.5)

(i) Let $F = \{\begin{bmatrix} a-b & 0 \\ -a+b & 2a-2b \end{bmatrix} \mid a, b \in R\}$. Then $F =$

a) $Span\{\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}\}$ ~~b)~~ $Span\{\begin{bmatrix} 4 & 0 \\ -4 & 8 \end{bmatrix}\}$ c) $Span\{\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & -2 \end{bmatrix}\}$ d) None of the previous statements

(ii) Let F as above and $D \in F$ such that D is not the zero matrix. Then Column space of D =

~~a)~~ R^2 b) R^4 c) $Span\{(1, -1)\}$ d) $Span\{(1, 0)\}$ e) Since different D has different columns, column space of D cannot be determined.

(iii) Given A is a 4×4 such that A is row-equivalent to $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 4 \end{bmatrix}$. Then $N(A) =$

a) $\{(x_2 + x_3, x_2, x_3, 0) \mid x_2, x_3 \in R\}$ b) $Span\{(1, 1, 1, 1), (0, 0, 0, 1)\}$ ~~c)~~ $Span\{(-1, 1, 0, 0), (-1, 0, 1, 0)\}$.
d) $\{(-x_2 + x_3, x_2, x_3, 0) \mid x_2, x_3 \in R\}$

(iv) Let A as in the previous question. Then one of the following points DOES NOT belong to the row space of A

a) $(-1, -1, -1, 5)$ b) (1, 1, 1, 0) ~~c)~~ (1, 1, -1, 4) d) (-1, -1, -1, 0)

(v) Let A as in question (ii). Given $A \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 6 \end{bmatrix}$ and $A \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 4 \end{bmatrix}$. Then the column space of A is

a) $span\{(0, 0, 0, 2), (-1, 0, 0, 1)\}$ b) $Span\{(1, 0, 0, 0), (1, 1, 2, 4)\}$ c) Span$\{(1, 0, 0, 0), (0, 1, 0, 0)\}$ ~~d)~~ span$\{(1, -1, -1, -1), (1, 0, 1, 3)\}$ e) More information is needed to find Column space of A.

(vi) Let $F = \{f(x) \in P_3 \mid f(-1) = 0\}$. Then F is a subspace of P_3. Hence $F =$

a) $Span\{6 + 6x, x^2 + 1\}$ b) span$\{x + x^2\}$ c) $Span\{x + x^2, 2x + x^2\}$ ~~d)~~ Span$\{x + 1, x^2 + 2x + 1\}$

(vii) One of the following is a basis for R^4

~~a)~~ $\{(4, 6, 0, 2), (-2, 8, 2, 2), (-4, -6, 3, 7), (-2, -3, 0, 10)\}$ b) $\{(1, 0, 0, 0), (1, 1, 0, 1), (0, 1, 0, 1), (0, 0, 0, 4)\}$
C) Any 4 points in R^4 is a basis for R^4. d) (a) and (b) and (c) will do

(viii) One of the following points belong to $Span\{(1, 1, 1), (-1, 1, 1), (3, -1, -1)\}$

a) $(4, 2\pi, 2\pi^+2)$ b) (0, 2, 3) c) $(1, \pi, -\pi)$ ~~d)~~ (1, 6, 6)

(ix) Let A be a 3×3 such that $(0, 4, 0) \in N(A)$, $A \begin{bmatrix} 1 \\ 8 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ 10 \end{bmatrix}$, and $A \begin{bmatrix} 0 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ 10 \end{bmatrix}$. Then $N(A) =$

a) $Span\{0, 1, 0)\}$ b) $Span\{(0, 4, 0), (1, 8, 0), (0, 7, 1)\}$ c) $Span\{(0, 4, 0), (1, 15, 1)\}$ ~~d)~~ $span\{(0, 1, 0), (1, 1, -1)\}$
e) More information is needed

QUESTION 3. (JUST WRITE T OR F, Total = 10 points)

(i) Let $F = \{A \in R^{2\times 2} \mid det(A) \leq 1\}$. Then F is a subspace of $R^{2\times 2}$

(ii) The span of any 5 polynomials in P_5 is equal to P_5

(iii) $F = \{g(x) \in P_4 \mid f(0) = 0$ or $f(1) = 0\}$ is a subspace of P_4

(iv) $D = \{f(x) \in P_3$ |f(0) = 1$\}$ is a subspace of P_3.

(v) It is possible to have 7 matrices in $R^{2\times 3}$ that are independent.

(vi) Every 6 points in R^5 are dependent

(vii) It is possible that the span of 6 points in R^3 is equal to R^3

(viii) If A is 3×3 and the system $AX = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$ has infinitely many solution, then it is possible that $Rank(A) = 1$

(ix) If A is a 3×5 matrix such that for every $B = (b_1, b_2, b_3) \in R^3$ the system $AX = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ has a solution, then dim(N(A)) = 2.

(x) $F = \{(b, -2a, 3 + b) \mid a, b \in R\}$ is a subspace of R^3

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Second Exam MTH 221 , Summer 011

Ayman Badawi

QUESTION 1. (Each = 1.5 points, Total = 12 points) Answer the following as true or false: NO WORKING NEED BE SHOWN.

(i) $dim(P_7) = 7$.

(ii) Every 5 points in R^4 are dependent

~~(iii)~~ Every 4 points in R^5 are independent

~~(iv)~~ Every 6 polynomials in P_6 form a basis for P_6

~~(v)~~ The point $(2, 2, 2, 10) \in span\{(1, 1, 1, 1)\ ,\ (-1, -1, -1, 7)\}$

(vi) If v_1, v_2 are independent points in R^3 and $v_3 \notin span\{v_1, v_2\}$, then $\{v_1, v_2, v_3\}$ is a basis for R^3

(vii) If v_1, v_2, v_3 are dependent points in R^5, then $v_1, -3v_1 + v_2, v_3$ are also dependent points in R^5.

(viii) If $D = span\{(2, -2, 0)\ ,\ (-2, 2, 1),\ (4, -4, 1)\}$, then $dim(D) = 2$

QUESTION 2. (Each = 2 points, Total = 28 points) Circle the correct letter for each of the questions below:

(i) One of the following statements is correct

~~a~~. $(2, 0, -2), (-2, 10, 20), (-4, -10, 1)$ are independent
b. (2, 1), (0, 1), 1,0) are independent
c. (0, 1, 1), (-1, 2, 0), (-1, 3, 1) are independent
d. $\{(2, 1), (1, 0.5)\}$ is a basis for R^2.

(ii) Let $F = \{(3a + b, 0, 6a + 2b) \mid a, b \in R\}$. We know that F is a subspace of R^3. Then $dim(F) =$

a. 2 b. 3 ~~c.~~ 1 d. cannot be determined

(iii) Consider F in the previous question. Then one of the following points does not belong to F.

a. (1, 0, 2) b. (2, 0, 4) ~~c.~~ (3, 0, 5) d. (0, 0, 0)

(iv) Given v_1, v_2, v_3 are independent points in R^{10}. One of the following statements is correct:

~~a.~~ $v_1, v_2, 2v_1 + v_3$ are independent points in R^3
b. $v_1, v_1 + v_3, v_3$ are independent points in R^3
c. $v_1, v_1 + v_2, -3v_1 + v_2$ are independent points in R^3
d. All previous statements are correct.

(v) Let $D = span\{(1, 1, 1, 1), (-1, -1, -1, 0), (0, 0, 0, 2), (-1, -1, -1, 4)\}$. One of the following is a basis for D

~~a~~. $B = \{(1, 1, 1, 1), (0, 0, 0, 1)\}$
b. $B = \{(1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 1)\}$
c. $B = \{(1, 1, 1, 1), (-1, -1, -1, 0), (0, 0, 0, 2)\}$
d. None of the previous is correct

(vi) Let $H = \{a + (b + 2c)x + (3b + 6c)x^2 \mid a, b, c \in R\}$ be a subspace of P_3. Then $dim(H) =$

a. 3 b. 1 c. 4 ~~d~~. 2

(vii) Let H as in the previous question. Then one of the following is a basis for H

a. $B = \{1, x, 3x^2\}$
b. $B = \{1, x + 3x^2, 3x + 6x^2\}$
c. $B = \{1, x, 2x, 3x^2, 6x^2\}$
~~d.~~ $B = \{1, x + 3x^2\}$.

(viii) Let $A = \begin{bmatrix} a_1 & 2 & 4 \\ a_2 & 4 & 8 \\ a_3 & -4 & -7 \end{bmatrix}$ such that det(A) = 20. The value of x_1 in solving the system $AX = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$ is

a. 5 b. 10 c. 0.2 d)Can not be determined

(ix) Consider the previous question, the value of x_2 in solving the system $AX = \begin{bmatrix} 4 \\ 8 \\ -7 \end{bmatrix}$ is

a. 0 b. 1 c. 0.1 d) None of the previous is correct

(x) Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 2 & 1 \\ -1 & -1 & -2 & 2 \\ -1 & -1 & -1 & -2 \end{bmatrix}$ the (3, 4)-entry of A^{-1} is

a. -3 b. -2 c. 0.2 d. 2 e. None of the previous is correct

(xi) Given A is a 2×2 matrix such that $\begin{bmatrix} 3 & 1 \\ 4 & -1 \end{bmatrix} + 2A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$. Then $A =$

a. $\begin{bmatrix} 1 & -1 \\ -5 & -4 \end{bmatrix}$ b. $\begin{bmatrix} 9 & 5 \\ -2 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 2 & -1 \\ -9 & 5 \end{bmatrix}$ d. None of the previous is correct

(xii) Given $\left(A^T \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right)^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$. Then $A =$

a. $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ b. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ c. $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$ d. None of the previous is correct

(xiii) One of the following is a subspace of R^3:

a. $\{(a, 2a - b^2, 0) \mid a, b \in R\}$ b. $\{(a + b, -2a, b) \mid a, b \in R\}$

c. $\{(3, -a, -b) \mid a, b \in R\}$ d. $\{0, ba + a, -2b) \mid a, b \in R\}$

(xiv) One of the following is a subspace of P_3:

a. $\{3 - ax + ax^2 \mid a \in R\}$ b. $\{3ax + ax^2 \mid a \in R\}$

c. $\{x + ax^2 \mid a \in R\}$ d. $\{a + ax + x^2 \mid a \in R\}$